RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

THIRD YEAR

B.A./B.SC. SIXTH SEMESTER (January – June) 2013

Mid-Semester Examination, March 2013

MATHEMATICS (Honours)

Date : 21/03/2013
Time : 11 am – 1 pm
Paper - VIII
Full Marks : 50

[Use Separate Answer Books for each Group]

Group - A

1. Answer **any three** questions : [3×5]

   a) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 3$, $2x + y = 1$ and $x - y = -1$. Find the line of action of the resultant.

   b) A variable system of forces in a plane has constant moments about two fixed points on the plane. Prove that their resultant passes through another fixed point on the plane.

   c) State and prove the principle of virtual work for any system of coplanar forces acting on a rigid body.

   d) Two heavy rings slide on a fixed smooth parabolic wire whose axis is horizontal and the rings are connected by a string which passes over a smooth peg at the focus. Prove that in the position of equilibrium the depths of the rings below the axis of the parabola are proportional to their weights.

   e) A smooth paraboloid of revolution is fixed with its axis vertical and vertex upwards; on it is placed a heavy elastic string of unstretched length $2\pi c$. When the string is in equilibrium show that it rests in the form of a circle of radius $4ac\pi\lambda/(4\pi a\lambda - cW)$, where W is the weight of the string, λ its modulus of elasticity and 4a, the latus rectum of the generating parabola.

Group - B

2. Answer **any two** questions : [2×2]

   a) Define 'Symbolic constants'.

   b) State the difference between break statement and continue statement.

   c) Show the output resulting from the following printf statement.

   ```
   float a = 2·5, b = 0·0005, c = 3000;
   printf ("%8·3f %8·3f %8·3f", a, b, c);
   ```

3. Answer **any two** questions : [2×3]

   a) Describe the output that will be generated by the following C program.

   ```
   # include <stdio.h>
    main ( )
    { int i = 0, x = 0;
      while (i<20)
      {if (i%5 = = 0)
          {x + = i;
            printf("%d",x);
        }
        ++i;
      }
     printf ("\n x = %d", x);
   }
   ```

b) Determine the output of the following :

```
# include <stdio.h>
  main ( )
  { int i = 0, x = 0;
     for (i=1; i<10; ++i)
     {if (i%2 = = 1)
           x + = i;
           else
           x – –;
            printf("%d",x);
            break;
        }
    printf ("\n x = %d", x);
}
```

c) A C program contains the following declaration and initial assignment
int i = 8, j = 5;
Determine the value of the following expression
2*((i/5)+(4*(j–3))%(i+j–2))+j/3;

Group – C

[Attempt either Unit –I or Unit – II)

Unit – I (Tensor Calculus)

Answer **any two** questions : [2×5]

4. a) Prove that $\frac{\partial A_r}{\partial x^s}$ is not a tensor even though A_r is a covariant tensor of rank one. [3]

b) Prove that Kronecker delta is a tensor of rank two. [2]

5. a) If $\xi(p,q)$ is the cofactor of A_{pq} in the determinant $d=\left|A_{pq}\right|\neq 0$ and $A^{pq}=\frac{\xi(p,q)}{d}$ then prove that $A_{pq}A^{rq}=\delta^r_p$. [3]

b) If A^{pq}_r and B^{pq}_r are tensors then prove that $A^{pq}_r - B^{pq}_r$ is a tensor. [2]

6. a) If a tensor A_{ijkl} is symmetric in the first two indices from the left and skew-symmetric in the second and fourth indices from the left then show that $A_{ijkl}=0$. [3]

b) Define with example the inner product of two tensors. [2]

Unit – II (Differential Geometry)

4. State the Frenet equations of a space curve. Compute curvature of the circular helix $\alpha:\mathbb{R}^3\to\mathbb{R}$ by [1+2]
$\alpha(s)=\left(a\cos\frac{s}{\sqrt{a^2+b^2}},\ a\sin\frac{s}{\sqrt{a^2+b^2}},\ \frac{bs}{\sqrt{a^2+b^2}}\right)$ with $a,b\neq 0$.

5. State definition of surface. Show that Torus of revolution is a surface. [1+2]

6. Let S be a surface, $P\in S$ and $X:U\to S$ a parametrization of S with $P\in X(U)$. Then show that— [4]
$T_pS=(dx)_{X^{-1}(P)}(\mathbb{R}^2)$

ꕤ